Analysis of Periodic Ferrite Slab Waveguides by Means of Improved Perturbation Method

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Abstract—**Periodic ferrite slab waveguides are analyzed by means of an improved perturbation method, and nonreciprocal leakage phenomena are shown theoretically. As an application of these phenomena, new planar isolators and circulators are proposed. Numerical examples are also provided.**

I. Introduction

RECENTLY, periodic structures created on thin-film waveguides have found many applications, such as beam couplers, filters, distributed feedback amplifiers, and lasers [1]–[5]. If a gyrotropic material such as a magnetized ferrite is included in a waveguide as a film and/or a substrate, the structure could exhibit nonreciprocal effects. When a grating structure is incorporated in such a waveguide, interesting phenomena can be exhibited.

In this paper we first provide an analysis of ferrite loaded open waveguides with periodic perturbations by means of an improved perturbation method which has been initially developed by Handa *et al.* [6], for analyzing periodically modulated waveguides made of isotropic materials and predict existence of nonreciprocal leakage phenomena. We then propose new planar isolators and circulators as applications of these phenomena.

We will perform an analysis of uniform ferrite loaded slab waveguides in Section II because the result is used in the analysis of periodically modulated ferrite slab waveguides in Section III. Section IV deals with the nonreciprocal leakage mechanism and describes how this mechanism is used for possible development of new nonreciprocal devices. Numerical examples will be provided in Section V. Finally, some discussions and conclusions are provided in Section VI.

II. Analysis of Uniform Ferrite Loaded Slab Waveguide

The improved perturbation technique for the grating structures requires the solution of the unperturbed waveguide. Fig. 1(a) shows an unperturbed waveguide corresponding to the grating structure in Fig. 1(b). The material constants $[\mu_g]$ and ϵ_g for $0<z<t_g$ in Fig. 1(a) are the volume-average values of the constants of the ferrite and air in the grating regions $0<z<t_g$ in Fig. 1(b). Use of $[\mu_g]$ and ϵ_g derived in this manner makes the present perturbation analysis much more accurate for especially thick gratings [6], because in conventional perturbation methods, the corrugation teeth of the grating are treated as incremental addition to a two layered structure consisting of the dielectric (ϵ_s, μ_0) and the ferrite $(\epsilon_f, [\mu_f])$. Obviously, the conventional perturbation method is not very accurate for gratings with a large thickness t_g.

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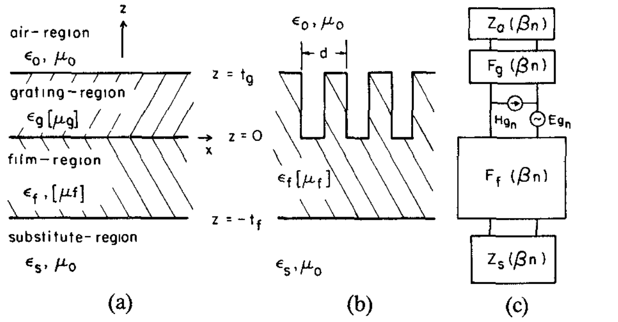


Fig. 1. Periodic ferrite slab waveguides. (a) Unperturbed system (uniform waveguide). (b) Perturbed system (periodic waveguide). (c) Equivalent circuit for nth space harmonics.

We assume that the structure and the excitation source are uniform and infinite in extent in the y direction. Hence, the fields are invariant with respect to the y direction, i.e., $\partial/\partial y=0$. A static magnetic field H_{dc} is applied in the y direction. Therefore, the tensor permeability of the magnetized ferrite will be of the form

$$[\mu]=\begin{bmatrix} \mu & 0 & j\kappa \\ 0 & \mu_0 & 0 \\ -j\kappa & 0 & \mu \end{bmatrix}.$$

We will treat only the TE mode, because the TM mode cannot exhibit nonreciprocity in the waveguide structure. We will also assume for the convenience of description that a time dependence $\exp(-j\omega t)$ is suppressed and all the materials are lossless, i.e., ϵ, μ, and κ are real numbers.

From the Maxwell's equations, we can derive the following wave equation for the E_y component [7]:

$$\left(\frac{d^2}{dz^2}+\omega^2\mu_e\epsilon-\beta^2\right)E_y=0 \tag{1}$$

where $\mu_e=(\mu^2-\kappa^2)/\mu$ is called the effective permeability. Here the x dependency is assumed to be $\exp(j\beta x)$ with the propagation constant β.

Equation (1) is easily solved for E_y. Moreover, the magnetic component H_x can also be derived from the

Maxwell's equation. We now define the transfer matrix of ferrite slab as

$$\begin{bmatrix} E_y \\ H_x \end{bmatrix}_{z=0} = \begin{bmatrix} F_f \end{bmatrix} \begin{bmatrix} E_y \\ H_x \end{bmatrix}_{z=-t_f} = \begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix} \begin{bmatrix} E_y \\ H_x \end{bmatrix}_{z=-t_f} \tag{2}$$

$$\begin{bmatrix} F_f \end{bmatrix} = \begin{bmatrix} \cos\theta_f + \dfrac{\sigma_f \beta}{k_f}\sin\theta_f & -j\dfrac{\omega\mu\epsilon_f}{k_f}\sin\theta_f \\ \dfrac{\omega^2\mu\epsilon_f - \beta^2}{j\omega\mu k_f}\sin\theta_f & \cos\theta_f - \dfrac{\sigma_f\beta}{k_f}\sin\theta_f \end{bmatrix} \tag{3}$$

where

$$\theta_f = k_f t_f \tag{4}$$

$$k_f = \sqrt{\omega^2 \mu_e \epsilon_f - \beta^2} \tag{5}$$

$$\sigma_f = \kappa/\mu. \tag{6}$$

The suffix f in the above expressions means "film." Similarly, the transfer matrix of the region, $0<z<t_g$, is expressed as follows:

$$\begin{bmatrix} E_y \\ H_x \end{bmatrix}_{z=0} = \begin{bmatrix} F_g \end{bmatrix} \begin{bmatrix} E_y \\ H_x \end{bmatrix}_{z=t_g} = \begin{bmatrix} A_g & B_g \\ C_g & D_g \end{bmatrix} \begin{bmatrix} E_y \\ H_x \end{bmatrix}_{z=t_g}$$

$$\begin{bmatrix} F_g \end{bmatrix} = \begin{bmatrix} \cos\theta_g - \dfrac{\sigma_g \beta}{k_g}\sin\theta_g & \dfrac{j\omega\mu\epsilon_g}{k_g}\sin\theta_g \\ -\dfrac{\omega^2\mu_g\epsilon_g - \beta^2}{j\omega\mu_g k_g}\sin\theta_g & \cos\theta_g + \dfrac{\sigma_g\beta}{k_g}\sin\theta_g \end{bmatrix}. \tag{7}$$

It should be noticed that the determinants of the transfer matrices, $|F_f|$ and $|F_g|$, are unity and diagonal components are real; whereas, off-diagonal components are pure imaginary. Hence, the ferrite slabs are reciprocal with respect to the z direction and are lossless.

Let us derive the characteristic equation for the unperturbed waveguide in Fig. 1(a) based on the transverse resonance technique. To this end, we first recognize that use of F-matrices makes it possible to represent these respective regions in terms of equivalent transmission lines. Next, both the substrate $(z<-t_f)$ and air $(z>t_g)$ regions are semi-infinite half spaces and, hence, can be replaced with semi-infinitely long transmission lines. The input impedances of these semi-infinite lines at $z=-t_f$ and $z=t_g$ are

$$Z_s = \left.\frac{E_y}{H_x}\right|_{z=-t_f} = \frac{j\omega\mu_0}{-k_s'} \tag{8}$$

$$Z_a = -\left.\frac{E_y}{H_x}\right|_{z=t_g} = \frac{j\omega\mu_0}{-k_a'} \tag{9}$$

where

$$k_s' = \sqrt{\beta^2 - \omega^2\mu_0\epsilon_s} \tag{10}$$

$$k_a' = \sqrt{\beta^2 - \omega^2\mu_0\epsilon_0}\,. \tag{11}$$

Now, from (3), (7), and (8), we can derive the input impedance at $z=t_g$ looking down the negative z direction and this impedance must be equal to Z_a given by (9). This process results in the characteristic equation

$$f(\beta) \equiv (A_f Z_s + B_f)(-C_g Z_a + D_g) - (C_f Z_s + D_f)(-A_g Z_a + B_g) = 0. \tag{12}$$

Solving this equation, we obtain the propagation constant in the layered waveguide. Because of the form of the characteristic equation, $-\beta$ is not necessarily a solution even if β is a solution. After the characteristic equation is solved, field distribution and power flow in the x direction can be easily calculated.

III. Analysis of Ferrite Loaded Slab Waveguides With Periodic Perturbation

It has been demonstrated that the improved perturbation method [6] approaches the accuracy obtainable from more rigorous treatments [1] while retaining the simplicity of perturbation methods. We extend this method to the structure containing gyrotropic materials.

The first step is to expand the fields in the perturbed system as follows:

$$\nabla \times (\mathbb{E} + \mathbb{E}^p) = j\omega\{[\mu] + [\mu^p]\}(\mathbb{H} + \mathbb{H}^p) \tag{13}$$

$$\nabla \times (\mathbb{H} + \mathbb{H}^p) = -j\omega\{\epsilon + \epsilon^p\}(\mathbb{E} + \mathbb{E}^p) \tag{14}$$

where superscript p means perturbation quantity. Neglecting all higher order terms, we have

$$\nabla \times \mathbb{E}^p = j\omega[\mu]\mathbb{H}^p + j\omega[\mu^p]\mathbb{H} \tag{15}$$

$$\nabla \times \mathbb{H}^p = -j\omega\epsilon\mathbb{E}^p - j\omega\epsilon^p\mathbb{E}. \tag{16}$$

We now recognize that the perturbed fields, $\mathbb{E}^p$ and $\mathbb{H}^p$, can be thought of as responses in the unperturbed system with the equivalent sources, $j\omega[\mu^p]\mathbb{H}$ and $j\omega\epsilon^p\mathbb{E}$.

Since (15) and (16) contain sources that vary periodically along x, $\mathbb{E}^p$ and $\mathbb{H}^p$ can be expanded in terms of space harmonics

$$\mathbb{E}^p = \sum_{n\neq 0} \mathbb{E}_n(z)\exp(j\beta_n x) \tag{17}$$

$$\mathbb{H}^p = \sum_{n\neq 0} \mathbb{H}_n(z)\exp(j\beta_n x) \tag{18}$$

where $\beta_n = \beta_0 + (2\pi/d)n$, d is the period of grating and β_0 is the propagation constant of the unperturbed surface wave. The zeroth terms are omitted in these summations, because they correspond to the unperturbed fields which are already extracted.

Material constants in the grating region vary periodically with the period of d, so that $[\mu^p]$ and ϵ^p can be expanded in Fourier series

$$[\mu^p]=\sum_{n\neq 0}[\mu_n^p]\exp\left(j\frac{2\pi}{d}nx\right) \tag{19}$$

$$\epsilon^p=\sum_{n\neq 0}\epsilon_n^p\exp\left(j\frac{2\pi}{d}nx\right). \tag{20}$$

The zeroth terms are also omitted because they are the volume averaged values $[\mu_g]$ and ϵ_g which have been used for establishing the unperturbed structure in Fig. 1(a).

A substitution of (17)–(20) into (15) and (16) yields the following equations:

$$\left(\frac{d^2}{dz^2}+\omega^2\mu_e\epsilon-\beta_n^2\right)E_{yn}(z)=F_n(z) \tag{21}$$

where

$$F_n(z)=-\omega^2\mu_e\epsilon_n^pE_y(z)-\frac{\beta_n}{\mu}\xi_nH_x(z)-\frac{\beta_n}{\mu}\zeta_nH_z(z)$$

$$-j\frac{\omega}{\mu}\zeta_n\frac{d}{dz}H_x(z)+j\frac{\omega}{\mu}\xi_n\frac{d}{dz}H_z(z)$$

$$\zeta_n=\mu_n^p\mu-\kappa_n^p\kappa$$

$$\xi_n=j(\mu_n^p\kappa-\kappa_n^p\mu). \tag{22}$$

In general, $E_{yn}(z)$ is a solution of the inhomogeneous equation given by (21). However, in the film, substrate, and air regions, $F_n(z)$ vanishes and the equation becomes a homogeneous one in these regions. Therefore, a treatment similar in Section II can be used for these regions.

In the film region, we can also define the transfer matrix of ferrite film for the nth space harmonics

$$\begin{bmatrix}E_{yn}\\H_{xn}\end{bmatrix}_{z=0}=\begin{bmatrix}F_{fn}\end{bmatrix}\begin{bmatrix}E_{yn}\\H_{xn}\end{bmatrix}_{z=-t_f} \tag{23}$$

where $F_{fn}=F_f|_{\beta=\beta_n}$.

In the substrate and air regions, the wave impedances for the nth space harmonics can be defined

$$Z_{sn}=\frac{j\omega\mu_0}{-k'_{sn}}$$

$$Z_{an}=\frac{j\omega\mu_0}{-k'_{an}}$$

where

$$k'_{sn}=\sqrt{\beta_n^2-\omega^2\mu_0\epsilon_s}$$

$$k'_{an}=\sqrt{\beta_n^2-\omega^2\mu\epsilon_0}\,.$$

In the grating region, the transfer-matrix and the distributed equivalent source are defined. After some mathematical manipulations, we have

$$\begin{bmatrix}E_{yn}\\H_{xn}\end{bmatrix}_{z=0}=\begin{bmatrix}F_{gn}\end{bmatrix}\begin{bmatrix}E_{yn}\\H_{xn}\end{bmatrix}_{z=t_g}+\begin{bmatrix}E_{gn}\\H_{gn}\end{bmatrix} \tag{24}$$

where $F_{gn}=F_g|_{\beta=\beta_n}$ is the transfer matrix. The last term in (24) is considered the equivalent source which is

$$\begin{bmatrix}E_{gn}\\H_{gn}\end{bmatrix}=-\begin{bmatrix}F_{gn}\end{bmatrix}\begin{bmatrix}E_n\\H_n\end{bmatrix}_{z=t_g}+\begin{bmatrix}E_n\\H_n\end{bmatrix}_{z=0}$$

$$E_n(z)=F_n(z)/(\beta_0^2-\beta_n^2)$$

$$H_n(z)=\left(\sigma_gk_{gn}E_n-\frac{d}{dz}E_n\right)/j\omega\mu_{eg}+(\xi_nH_z-\zeta_nH_x)/\mu_{eg}\mu_g$$

and

$$k_{gn}^2=\omega^2\mu_{eg}\epsilon_g-\beta_n^2.$$

From the discussions above, we can derive an inhomogeneous equation for each space harmonic

$$\begin{bmatrix}E_{yn}\\H_{xn}\end{bmatrix}_{z=0}=\begin{bmatrix}F_{gn}\end{bmatrix}\begin{bmatrix}E_{yn}\\H_{xn}\end{bmatrix}_{z=t_g}+\begin{bmatrix}E_{gn}\\H_{gn}\end{bmatrix}$$

$$=\begin{bmatrix}F_{fn}\end{bmatrix}\begin{bmatrix}E_{yn}\\H_{xn}\end{bmatrix}_{z=-t_f}. \tag{25}$$

We can find the fields of the nth space harmonics from the following equations:

$$H_{xn_{z=t_g}}=\{(-C_{gn}Z_{an}+D_{gn})E_{gn}-(-A_{gn}Z_{an}+B_{gn})H_{gn}\}/f(\beta_n) \tag{26}$$

$$H_{xn_{z=-t_f}}=\{(C_{fn}Z_{an}+D_{gn})E_{gn}-(A_{fn}Z_{sn}+B_{fn})H_{gn}\}/f(\beta_n) \tag{27}$$

where $f(\beta)$ is defined in (12). The equivalent circuit for the nth space harmonics is given in Fig. 1(c).

When k_{sn}^2 is positive, the nth space harmonics will leak away into the substrate region; whereas, when k_{an}^2 is positive, a leakage into the air region will occur. Power flows into the substrate and air regions are given, respectively, as follows:

$$P_{sn}=Z_{sn}|H_{xn}|^2_{z=-t_f} \tag{28}$$

$$P_{an}=Z_{an}|H_{xn}|^2_{z=t_g}. \tag{29}$$

Once we know the P_{sn}'s and P_{an}'s for all leaky harmonics, we can calculate the attenuation constant due to these leakages

$$\alpha=\sum(P_{sn}+P_{an})/2Pd. \tag{30}$$

The summation in (30) is carried out for the terms for which $k_{sn}'^2$ and/or $k_{an}'^2$ are positive. P means a power flow in the x direction of surface wave.

IV. Nonreciprocal Leakage Phenomenon and Its Application to Isolators and Circulators

Owing to the anisotropy of magnetized ferrite, a propagation constant of surface wave traveling along the $+x$ direction is different from the one for the backward travel-

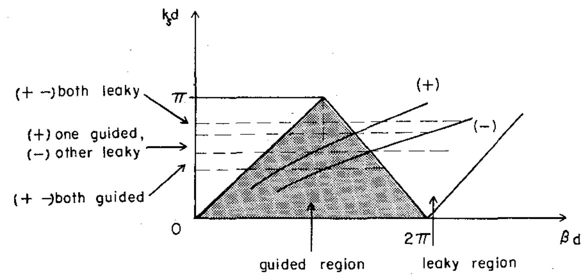
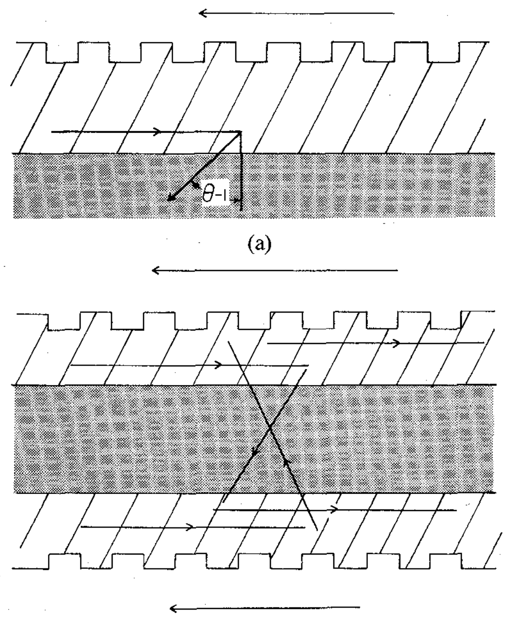


Fig. 2. Brillouin diagram for periodic structures.



(a)

(b)

Fig. 3. Grating type isolator and circulator.

ing wave, i.e., $\beta^+ \neq \beta^-$. Therefore, we can find a possibility that, if the grating period d is properly chosen, nonreciprocal leakage will occur. For instance, we can make the -1st space harmonic in the negative direction leaky while the one into the positive direction is not leaky by choosing

$$\beta^+ - \frac{2\pi}{d} < -\beta_s < \beta^- - \frac{2\pi}{d} \tag{31}$$

where $\beta_s = \omega\sqrt{\mu_0 \epsilon_s}$. By referring to the $k-\beta$ diagram as depicted in Fig. 2, we see clearly that only the -1st space harmonics for the backward wave falls into the leaky region.

The nonreciprocal leaky mechanism attainable by choosing parameters to satisfy (31) may be used to develop an isolator. This planar isolator is similar in its function to a field displacement type [8], and the grating etched at the surface of ferrite film can act as a loss mechanism. However, unlike the field displacement type, the grating introduces no insertion loss for the backward case. This is an important characteristic of this isolator. In contrast, a conventional field-displacement type isolator uses a resistive film so that insertion loss is inevitable.

Furthermore, if two grating ferrite waveguides are placed parallel as shown in Fig. 3, 4-port circulator action can be expected. Nonreciprocal leakage and coupling mechanism are used in this circulator.

V. Numerical Results

According to the method described in this paper, numerical calculations have been done and examples of the results are shown in Figs. 4 and 5. Fig. 4 shows the leakage loss for the reciprocal waveguide to check the validity of this theory. We developed a computer program for nonreciprocal cases and ran it by supplying isotropic parameters for $[\mu]$. It is seen from figures that from 99 to 101 GHz only the wave propagating in the negative direction undergoes attenuation. Available frequency bandwidths are about 2 GHz for both cases, but attenuation constants for 0.2-mm grating depth are about ten times the ones for 0.1 mm. For these cases, relatively low saturation magnetizations and applied magnetic fields are chosen. However, if the magnetizations and/or the magnetic fields are strengthened, the bandwidth will be increased.

VI. Discussions and Conclusions

It should be noticed that the perfect isolator action (infinite isolation and no insertion loss) proposed in the previous section may be obtained only if the length of

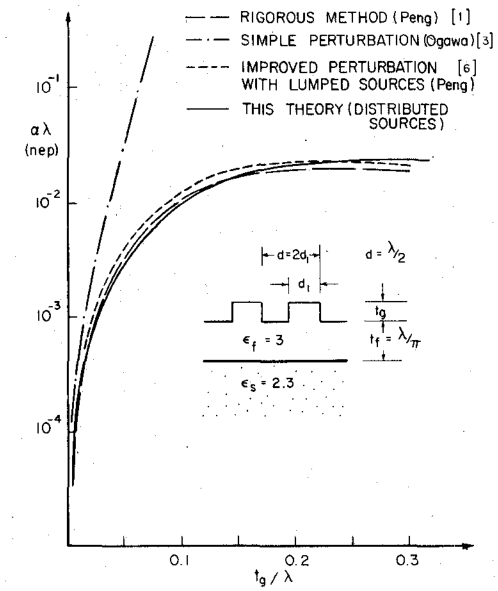


Fig. 4. Leakage loss for periodic dielectric waveguide.

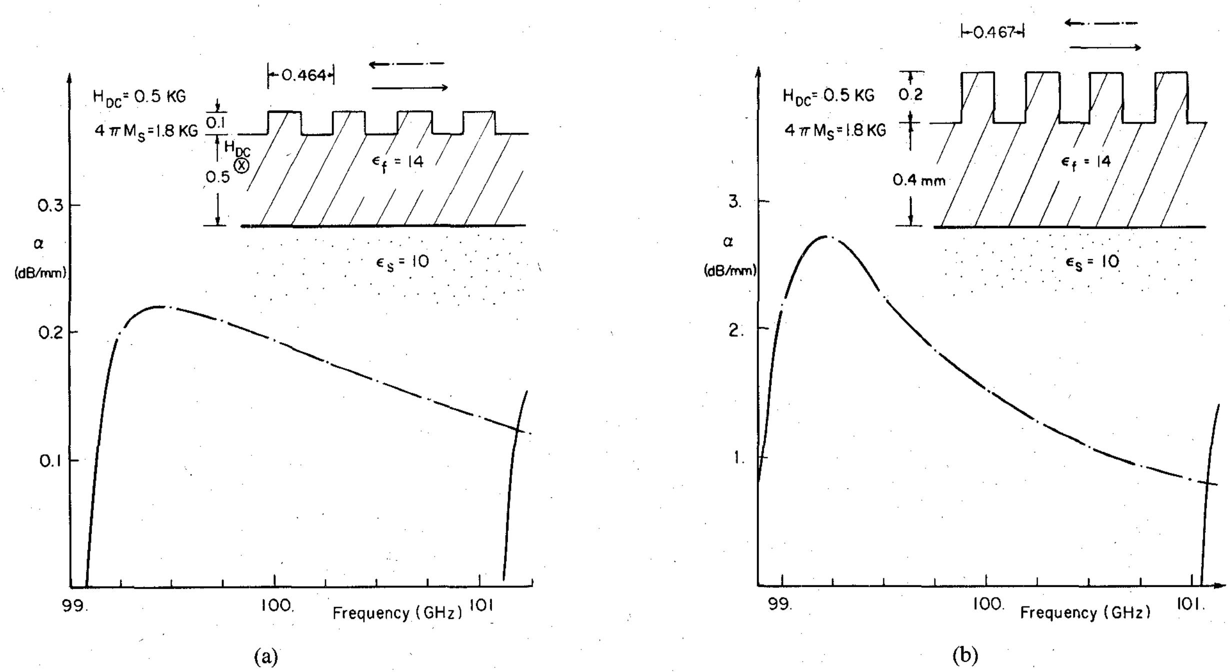


Fig. 5. Nonreciprocal leakage loss. (a) $t_g = 0.1$ (mm). (b) $t_g = 0.2$ (mm).

grating section is infinitely long. In practive, the length is necessarily finite. In such cases, several considerations are needed to improve the performance of the device. For instance, we may use tapered distributions on the amount of perturbations of grating elements.

In the present analysis, the coupling between any two space harmonics as well as the reflection [9] at the input port are not taken into account. These phenomena will degrade performance of the nonreciprocal devices.

Bandwidth of this type of isolator is proportional to $\Delta\beta = \beta^- - \beta^+$ which also depends on κ, the anisotropy of the magnetized ferrite. At millimeter and submillimeter frequencies, we can obtain only a very small anisotropy. Therefore, we must modify the waveguide structure to enhance $\Delta\beta$. One possible method would be to employ a weaky guiding concondition [10]. However, the tolerance of d becomes severe, and material losses would not be neglected.

Furthermore, a more efficient nonreciprocal leakage will be achieved by making a grating at the bottom face of the ferrite film because the field distribution suffers greater displacement towards the substrate region for the backward direction.

Periodic ferrite slab waveguides are analyzed by using an improved perturbation theory and the nonreciprocal leakage phenomenon is shown theoretically and numerically.

As an application, new types of planar isolators and circulators are proposed. Some of the problems of actual implementations are discussed.

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Fundamental Considerations in Millimeter and Near-Millimeter Component Design Employing Magnetoplasmons

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***Abstract*—The feasibility of using surface magnetoplasmons on semiconducting substrates to obtain circuit functions which match those of ferrite loaded devices at lower frequencies, is investigated. This article describes some initial results obtained in our study of performance characteristics using the best loss parameters available for GaAs materials. Canonical models are considered which relate directly to proposed configurations for differential phase shifters and isolators in the millimeter and near-millimeter ranges.**

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I. Introduction

An INVESTIGATION is presented on the feasibility of using the properties of surface magnetoplasmons on semiconducting substrates as a basis for developing

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